

Math 211 - Bonus Exercise 13 (please discuss on Forum)

- 1) Write down matrices that describe the action $S_n \curvearrowright \mathbb{R}^n$ given by permuting the coordinates, i.e.

$$\sigma \cdot (x_1, \dots, x_n) = (x_{\sigma^{-1}(1)}, \dots, x_{\sigma^{-1}(n)})$$

Side question: why did I have to write σ^{-1} and not σ in the formula above?

- 2) Find (proper) irreducible subrepresentations $V, V' \subset \mathbb{R}^n$ (with respect to the action of S_n from the previous problem), such that $V \oplus V' = \mathbb{R}^n$.

- 3) Suppose you have a representation $G \curvearrowright V$ on a real vector space V . Can you use this representation to produce a function $f : G \rightarrow \mathbb{R}$ which is constant on conjugacy classes?

- 4) Remember the left action of a group G on itself. It's not a representation because G is not a vector space, but can you “upgrade” it so that it becomes a representation?